DETERMINATION OF SYSTEM FEASIBILITY AND VIABILITY EMPLOYING A JOINT PROBABILISTIC FORMULATION

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Abstract

The present paper outlines a method for probabilistic multi-criteria decision making. Recognizing the limitations of traditional probabilistic methods in accounting for multiple decision criteria in conceptual or preliminary design, this new method combines probabilistic treatment of uncertain information with a multi-criteria decision making technique. The paper describes how the method addresses a need in Multi-Disciplinary Optimization and Analysis as well as the advanced technology selection process in conceptual and preliminary design. The mathematical foundations of a general joint probabilistic formulation are outlined. Two specific functions are introduced that compute the joint probability: the joint empirical distribution function and the joint probability model. The utility of both functions is demonstrated in a proof of concept study for two criteria, applying both functions to a challenging aircraft design problem, the High Speed Civil Transport. This example application addresses two pressing issues: the identification of a feasible design space for a given design concept and the evaluation of viability of a given aircraft design. Finally, the advantages and limitations of the empirical distribution function method as well as the joint probability model are summarized.

Keywords: Aerospace Systems Design, Probabilistic Design, Multi-Criteria Decision Making, Joint Probability, Feasibility, Viability, Multi-Disciplinary Optimization/Analysis

Definitions

Attribute: System characteristic that denotes or quantifies its production schedule and cost or operational behavior, e.g. life cycle cost, gross weight, or excess power.

Constraint: Relationship that must be satisfied, arising from physical laws and limitations or from compatibility conditions on individual variables.

Criterion: Measure of effectiveness that is the basis for an evaluation;[1] attributes and constraints become criteria if the decision is based on their outcome.

Cumulative distribution function (CDF): Relationship between criteria values and their cumulative probability (probability of achieving criteria values less than or equal to the one specified).

Random variable: Variable whose values cannot be predicted with certainty but instead only with an associated probability.[2]

Mean (μ): Expected value of a random variable, determined by the weighted average of all its possible values.

Standard deviation (σ): Measure of dispersion or variability of random variable values, determined by their deviation from the mean.

Covariance: Measure of the degree of (linear) interrelationship between values of two random variables.

Correlation factor (ρ): Normalized covariance of two random variables; indicates mathematical dependence between two criteria.

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Introduction and Motivation

Multi-disciplinary analysis and design optimization (MDA/MDO) has frequently been formulated as a deterministic process that is based on a design objective with an associated set of design parameters, variation of which yields an optimal value for the objective. Several objective functions have been proposed,[3,4] most of which are for deterministic processes and only few seem to reflect the decision making process. The optimization process is being complicated further due to design parameters arising from a multitude of interdependent disciplines. Addressing those interactions and the resulting optimization complications has been the founding work of the MDO community. It is the authors' contention that the design process must not only address interactions between traditional aerospace disciplines (e.g. aerodynamics, structures, controls, propulsion), but should also account for "life cycle" disciplines (e.g. economics, reliability, manufacturability, safety, supportability, etc.). A major problem that arises with regards to these 'new' disciplines is that they are often characterized by incomplete or uncertain information impeding the performance and economic viability predictions. The variety in types of uncertainty, ranging from modeling and simulation errors to economic forecasts to expert opinion on new technologies, makes the modeling of information even more difficult. One modeling solution pursued by the authors is a probabilistic treatment of the uncertain information, yielding an approach to *probabilistic design process.*

The emergence of "life cycle" disciplines has mainly been motivated by the current paradigm shift to design for affordability from design for performance. This paradigm shift requires the design and evaluation of a system to be no longer dictated solely by mission capability requirements, but rather by a desire to balance mission capability, system effectiveness, and cost. The consequential trade-off between benefit and cost is the main foundation of design for affordability. The benefits as perceived by the customer can have such attributes as increased performance, reduction in cycle time, improvements in product safety, enhanced dependability, etc. The metric used for cost depends largely on the application, but is usually represented by overall life cycle cost, investment, or a commonly used criterion for commercial transports, the required average yield per revenue passenger mile.

A key problem in complex systems design is measuring the 'goodness' of a design, i.e. finding a criterion through which a particular design is determined 'best'. Traditional choices such as gross take-off weight, acquisition cost, and payload individually fail to fully capture the life cycle characteristics of a system. Thus, a common approach has been to combine all criteria together into one equation termed the overall evaluation criterion, OEC. This equation is often very simple in its mathematical structure due to lack of any better model for the decision process. Recognizing this lack of proper decision process modeling, the authors propose an approach in this paper that uses aircraft attributes, such as take-off gross weight, acquisition cost, payload, safety, and supportability, concurrently as *decision criteria* for the evaluation of designs. This evaluation is not based on a summation of criteria, as with an OEC, but rather the probability of satisfying all criteria at the same time, a notion similar to a Pareto-optimality.[§] The main difference with respect to Pareto-optimality lies in the optimizable objective function, called probability of success (of satisfying all criteria).

This concurrent, multi-criteria approach lends itself more suitably to aircraft design than a single-criterion approach since customers typically like to see <u>all</u> decision criteria satisfied. For example, a probabilistic multi-criteria approach can yield the design solution which maximizes the probability of low cost, high capacity, speed, and dependability, while a single objective design will only yield an optimum in one of these criteria, neglecting all others.

Another issue is the modeling and use of uncertain information intrinsic to system descriptions in conceptual and preliminary design. For example, the designer may have a flight path or mission scenario for the aircraft, but is unclear about the operating conditions. One modeling option is to treat this incomplete information probabilistically, accounting for the fact that certain values may be prominent, while the actual value during operation is unknown. By assigning probability estimates to the values within the range of interest, the method guarantees that all values are kept as possible solutions. Using these values and their corresponding likelihood as design assumptions, virtual systems can be obtained whose operational characteristics are probabilistic in nature. In other words, a probabilistic design method yields the aircraft's attributes, and thus the decision criteria, as random variables.

If multiple, interdependent criteria are needed for the decision making, a joint-probabilistic formulation is needed to accurately estimate the probability, since the marginal, or univariate, distribution for each criterion does not indicate the likelihood of any other criterion value. In many cases aircraft attributes are in fact interdependent, since they are evaluated by the same design process or analysis. For example, the probability of cost being below a particular value depends on the value of capacity, speed, and system

[§] State of economic affairs where no one can be made better off without simultaneously making another worse off.[5]

reliability. The proposed joint-probabilistic approach to multi-criteria aircraft design will facilitate precisely this estimate.

A further use of the method, proposed in this paper, is the evaluation of concept or system feasibility. In conceptual design many different design solutions are evaluated as to how well they meet their objective. Among the most important requirements are system feasibility and viability, i.e. does the concept satisfy all design constraints and customer requirements, and if not, by how much are they violated. Hence, it is of fundamental importance to have an evaluation method at hand that determines feasibility and viability rapidly for many different design concepts. The method proposed here satisfies this need, as explained further in the proof of concept section of this paper.

Multi-Criteria Decision Making and Multi-Disciplinary Optimization

The techniques called for in the previous section fall under Multiple Criteria Decision Making (MCDM), which refers to making decisions in the presence of multiple, usually conflicting criteria[1] and attempts to facilitate the decision making process that is carried out by the decision maker. Hwang and Yoon[1] point out that MCDM can be classified into two categories: Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). The distinguishing feature of MADM is the selection of the best of a finite number of design solution alternatives, while MODM selects from an infinite number of alternatives. In other words, MADM can be used for selection and MODM for design. The joint probabilistic decision making method introduced in this paper is intended to facilitate a decision making process involving attributes, objectives and constraints. Thus, it is the authors' contention that this method falls in the MCDM category directly, without further separating the criteria into attributes and objectives. The method can be used as a MADM as well as a MODM tool. Hence, the notation in this paper distinguishes between attributes and criteria. Attributes are aircraft characteristics which become criteria, if the decision depends on their outcome. In this setting, objectives are always criteria.

MCDM is particularly useful to the aircraft MDO problem, since it treats problems with a multitude of highly interdependent (disciplinary) criteria. Many disciplinary optimization approaches suffer from a selection problem that is inherent to the optimization process. Every optimization uses an objective function, whose value is to be optimized, and a model of the objective function that accurately reflects its dependency on the input parameters. The optimization process determines an optimal setting for the input parameters that yields the best objective function value achievable. One of the problems the MDO community has been facing is the selection of the objective function. It is suggested here that it is best not to combine the disciplinary attributes into one, but rather to make use of the joint probabilistic approach and maximize the probability of success. This probability satisfies all criteria concurrently, while making success, the ultimate customer desire, the objective function.

Multivariate Probability Theory

Since it was established that the aircraft design process is multi-disciplinary and design solutions are typically based on uncertain assumptions, aircraft are categorized by a multitude of criteria which are probabilistic in nature. To accommodate both aspects of design concurrently, an extension of the commonly used univariate probability theory is needed. It is insufficient to look at each criterion and its distribution independently, since all attribute values are generated by the same design process and are thus interdependent. The assumption of independent criteria is therefore typically unfounded. The aforementioned necessary extension is consequently a probability theory for *jointly distributed random variables*.

Definition: Let X_1, X_2, ….,X_n be a set of random variables defined on a (discrete) probability space Ω. The probability that the events $X_1=x_1$, $X_2=x_2$, ….,and $X_n=x_n$ happen *concurrently*, is denoted by $f(x_1,x_2,\ldots,x_n) = P(X_1=x_1, X_2=x_2, \ldots, X_n=x_n)$.[**] If the function $f(x_1,x_2,\ldots,x_n)$ is discrete, it is called the *joint probability mass function* of X_1, X_2, ….,X_n and has the following properties:[6]

$$
\begin{gathered}
0 \le f(x_1, x_2, \ldots, x_n) \le 1 \\
\sum_{(x_1,x_2,\ldots,x_n)} \sum_{\in \Omega} f(x_1,x_2,\ldots,x_n) = 1 \\
P[(X_1, X_2, \ldots, X_n) \in A] = \sum_{(x_1,x_2,\ldots,x_n)} \sum_{\in A} f(x_1,x_2,\ldots,x_n),\ A \subseteq \Omega
\end{gathered}
\tag{1}
$$

If $f(x_1,x_2,\ldots,x_n)$ is continuous, it is called *joint probability density function* of X_1, X_2, ….,X_n and has the following properties:[6]

$$
\begin{gathered}
0 \le f(x_1, x_2, \ldots, x_n) \\
\int_\Omega \cdots \int f(x_1,x_2,\ldots,x_n) dx_1 dx_2 \ldots dx_n = 1 \\
P[(X_1, X_2, \ldots, X_n) \in A] = \int_A \cdot\cdot \int f(x_1,x_2,\ldots,x_n) dx_1 dx_2 \ldots dx_n,\ A \subseteq \Omega
\end{gathered}
\tag{2}
$$

If the lower bound of *A*, the set of desired solutions, is equal to the infimum[††] of Ω for all X_i, i.e.

[**] We use the common notation: $P(X_1=x_1, X_2=x_2, \ldots, X_n=x_n) = P[(X_1=x_1) \cap (X_2=x_2) \cap \ldots \cap (X_n=x_n)]$.

[††] Greatest lower bound.

if A = (inf$_i$(Ω), a_i], for all i = 1,2,...,n, a function $F(a_1,a_2,\ldots,a_n)$ can be defined, such that:

$$F(a_1,a_2,..,a_n) = P[(X_1,X_2,..,X_n)\in A] = \sum_{(x_1,x_2,\ldots,x_n)}\cdots\sum_{\in A} f(x_1,x_2,..,x_n),\ A\subseteq\Omega \qquad (f \text{ is discrete}) \qquad (3)$$

$$F(a_1,a_2,..,a_n) = P[(X_1,X_2,..,X_n)\in A] = \int_A\cdots\int f(x_1,x_2,..,x_n)dx_1dx_2..dx_n,\ A\subseteq\Omega \qquad (f \text{ is continuous}) \qquad (4)$$

F is called the joint cumulative probability distribution function.[7] For $\Omega = \Re^n$‡‡ and a continuous function f:

$$F(a_1,a_2,..,a_n) = P[(X_1,X_2,..,X_n)\in((-\infty,-\infty,..,-\infty),(a_1,a_2,..,a_n))] = \int_{-\infty}^{a_n}\cdots\int_{-\infty}^{a_1} f(x_1,x_2,..,x_n)dx_1..dx_n \qquad (5)$$

We also use the common notation: $F(a_1,a_2,..,a_n) = P(X_1\le a_1, X_2\le a_2, \ldots, X_n\le a_n)$.

The univariate probability function f_{Xi} for each criterion X_i, obtained from the traditional probabilistic design process, can also be generated with the joint probability function f. f_{Xi} is called *marginal probability mass or density function* of X_i and is defined by:

$$f_{X_1} = \sum_{(x_2,\ldots,x_n)}\cdots\sum_{\in R} f(x_2,\ldots,x_n) \qquad (f \text{ is discrete}) \qquad (6)$$

$$f_{X_1} = \int_R\cdots\int f(x_2,\ldots,x_n)dx_2\ldots dx_n \qquad (f \text{ is continuous}) \qquad (7)$$

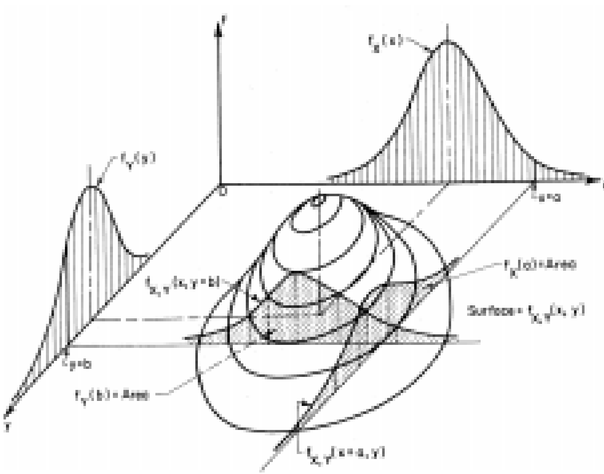

Figure 1: Joint and marginal PDF of continuous criteria X and Y[2]

To further illustrate the concept of joint probability, an example for two continuous criteria, X and Y, is displayed in Figure 1. The joint probability function, $f_{X,Y}(x,y)$, creates the surface of a probability 'hump' in the x-y-f-space, characterized by rings of constant probabilities. The distribution curves over the x- and y-axis are the aforementioned marginal probability functions $f_X(x)$ and $f_Y(y)$, respectively. Also displayed in Figure 1 are two 'cuts' through the probability 'hump', marking the probability distributions $f_{X,Y}(x=a,y)$ and $f_{X,Y}(x,y=b)$ and their respective areas underneath $f_X(a)$ and $f_Y(b)$.

The last necessary concept to mention here for the development of a joint probabilistic formulation is the concept of *independence of criteria*. Two random variables X and Y are said to be independent, if

$$f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y)\,, \qquad (8)$$

otherwise X and Y are said to be dependent. This dependence is a mathematical notion and may not be confused with 'causal dependence'. A simple example for mathematical dependence without causal dependence is the number of times a person takes an umbrella to work and the number of times he wears long pants in a given month. The two numbers increase <u>similarly</u> with the number of rainy days in that month, i.e. they are (mathematically) dependent. They are, however, <u>not causally</u> dependent, since wearing pants does not depend on taking an umbrella or vice versa, but rather on the rain the person has to face on the way to work.

From here on we will refer to mathematical dependence as correlation. Correlation is measured by the *covariance* of two criteria, X and Y, and defined by[2]

$$\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]. \qquad (9)$$

It is more convenient, however, to use a normalized covariance, called *correlation coefficient*:[2]

$$\rho = \frac{Cov(X,Y)}{\sigma_X\sigma_Y}. \qquad (10)$$

The correlation coefficient is defined over the interval [-1,1], indicating strong positively correlated criteria with values close to 1 and strong negatively correlated criteria with values close to –1. The criteria are independent, if $\rho = 0$. In aerospace systems design ρ can be quite difficult to calculate by Equation (10). It is much more effective to view the correlation coefficient differently for calculation purposes. Jointly collected data from a probabilistic or any other analysis can be thought of as vectors of numbers. The correlation coefficient measures the orthogonality, i.e. independence, of both vectors. ρ is simply the cosine of the angle between the two criterion vectors. It does not reflect any causal relationship, it merely indicates their alignment. For $\rho = 1$, vectors are parallel and point in same direction, for $\rho = -1$, vectors are parallel and point in opposite direction. For $\rho = 0$, vectors are orthogonal and the criteria are independent. The correlation coefficient plays a significant role in the formulation of joint probability distribution models as described in the next section.

‡‡ $\Re^n$ denotes the set of all real valued n-touples.

Probability Functions

Attention is now directed to the implementation of this probabilistic formulation into the design process. The necessary transition from the mathematical formulation above to a probabilistic model that yields the information relevant for multivariate decision making is described in this section.

Joint Probability Model: The first joint probability density function introduced here is an analytical probability model for criteria whose univariate distributions and their corresponding means and standard deviations are known. All necessary information for the model can be generated by the traditional probabilistic design process, using its output of univariate criterion distributions. A particular model for two criteria with normal distributions, represented by Equation (11), has been introduced by Garvey in Reference 8. Garvey generated further models for two criteria with combinations of normal and lognormal distributions, which are summarized in Reference 9. For the proof of concept study in this paper, only Equation (11) is used as a *joint probability model.*

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\{\frac{1}{2\rho^2-2}[(\frac{x-\mu_X}{\sigma_X})^2 - 2\rho(\frac{x-\mu_X}{\sigma_X})(\frac{y-\mu_Y}{\sigma_Y}) + (\frac{y-\mu_Y}{\sigma_Y})^2]\} \quad (11)$$

Note that the only information needed for this model consists of the means μ_X and μ_Y, the standard deviations σ_X and σ_Y, and the correlation coefficient ρ for the criteria X and Y.[§§] The model variables, x and y, are defined over the interval of all possible criterion values. The advantage of this model is the limited information needed, which makes it very flexible for use and application. For example, if only expert knowledge and no simulation/modeling is available in the early stages of design or technology development, educated guesses for the means, standard deviations, and correlation coefficients can be used to execute the joint probability model. It also lends itself to the use in combination with increasingly important fast probability integration techniques.[10,11,12,13,14,15]

Empirical Distribution Function: The second probability function to be introduced and used in this paper is the Empirical Distribution Function (EDF), named after the empirically collected data samples it is based on. The *univariate probability mass function* for a random variable X is defined for n samples as:

$$f_X(x) = \tfrac{1}{n}\sum_{i=1}^{n} I(a_i = x), \quad (12)$$

where $I(a_i = x) = \begin{cases} 1 & \text{for } a_i = x \\ 0 & \text{otherwise} \end{cases}$

a_i are the criterion sample values derived from a sampling method such as the Monte Carlo Simulation, while x is the criterion value of interest.

The *cumulative probability function* is consequently defined as:

$$F_X(x) = \sum_{\min(z)}^{x} f_X(z) = \tfrac{1}{n}\sum_{i=1}^{n} I(a_i \le x), \quad (13)$$

where $I(a_i \le x) = \begin{cases} 1 & \text{for } a_i \le x \\ 0 & \text{otherwise} \end{cases}$

Recognizing the joint probability notation from above, the univariate EDF can easily be extended to more random variables. The *joint probability mass function* can then be formulated as:

$$f(x_1, x_2, .., x_m) = \tfrac{1}{n}\sum_{i=1}^{n} I\left((a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m)\right) \quad (14)$$

and $I\left((a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m)\right) = \begin{cases} 1 & \text{for } (a_{i1}, a_{i2}, .., a_{im}) = (x_1, x_2, .., x_m) \\ 0 & \text{otherwise} \end{cases}$

Similarly, the *joint cumulative probability distribution function* can be formulated as:

$$F(x_1, x_2, .., x_m) = \tfrac{1}{n}\sum_{i=1}^{n} I(a_{i1} \le x_1, a_{i2} \le x_2, .., a_{in} \le x_n) \quad (15)$$

and $I(a_{i1} \le x_1, a_{i2} \le x_2, .., a_{in} \le x_n) = \begin{cases} 1 & \text{for } (a_{i1} \le x_1, a_{i2} \le x_2, .., a_{in} \le x_n) \\ 0 & \text{otherwise} \end{cases}$

The joint EDF depends on joint samples for the criteria only, and is not limited by any assumptions about criterion distributions made beforehand. It does not rely on any particular sampling method either and can be used as long as sample data is available. The need for this data, on the other hand, is its very limitation, since it can only be used in a design process with available simulation/modeling. Given enough sample data, however, the joint EDF yields the most accurate joint distribution prediction, since it does not rely on any normality assumption for the criterion or any approximation methods to generate the criterion statistics needed. Its greatest advantage yet lies in the missing requirement for a correlation coefficient, which can be difficult to estimate reliably in a design process. For very large numbers of sample data, the joint EDF can yield the exact solution for the joint distribution. However, in product design, a large number of process evaluations may not be a feasible option. The prediction accuracy of the Joint Probabilistic Model and joint EDF is in this case similar, which is why both functions have been introduced here and are executed for an example study in the next section.

[§§] The normality assumption for the attribute distributions is already part of the model, however, it may also be regarded as information about the output distribution that helps selecting the model.

Application Examples

High Speed Civil Transport Aircraft

The baseline aircraft used for these example studies is a notional High Speed Civil Transport (HSCT) depicted in Figure 2. The vehicle has an area-ruled fuselage (maximum diameter of 12 ft.), a double delta planform, and four nacelles below the wing, housing mixed flow turbofan (MFTF) power plants. The values for some of the important design parameters are given in Table I. The mission profile for this aircraft encompasses a split subsonic/supersonic mission which results from the restriction of subsonic flight over land, where the length of the subsonic cruise segment is assumed to be 15% of the design range.

Table I: Description of the Baseline HSCT

Parameter	Baseline
Range	5000 nm
Payload	300 Passengers
Fuselage length	310 ft.
Span	77.5 ft.
Inboard Sweep	74 deg.
Outboard Sweep	45 deg.
Wing Reference Area	9,000 ft^2
Mach Number	2.4
Supersonic Cruise Altitude	~63,000 ft.
Sustained Load	2.5 g

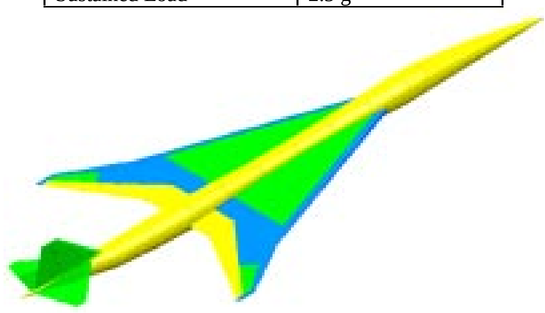

Figure 2: Notional HSCT

Finding an optimal configuration for a supersonic transport vehicle is a multi-disciplinary and quite difficult task. Choosing a wing planform shape, for example, is driven by the need for efficient performance at both sub- and supersonic cruise conditions, a conflicting design objective in itself.[16,17] Furthermore, the trades involved in planform selection are complicated by different discipline considerations for aerodynamics, structures, propulsion, etc., and the presence of design and performance constraints at the system level which are directly related to the wing. The limit on approach speed, for example, is mostly a function of wing loading. Fuel volume requirements impact the wing size and shape. Both become sizing criteria and are treated as constraints that tend to increase the wing in size. But increased wing area yields higher induced and skin friction drag, thus increasing fuel consumption, and so on. Additional design challenges are presented by takeoff and landing field length limitations (less than 10,500 ft) that are modeled as design constraints for the feasibility problem.

The Feasibility Problem

The first application example demonstrates the application of the joint probabilistic concept to the "determination of feasible space" problem. This problem is not a decision making problem in the strictest sense, however, it facilitates the design process, particularly the technology selection process (see References 11, 12, and 13). The aim is to determine as early as possible in the design phases whether a particular design *concept*, like the HSCT, has a chance of having a feasible design *solution*.[***] A *feasible design* is defined as a design which satisfies all imposed constraints. Typical constraints in aircraft design are limitations on the values for approach speed, landing and take-off field length, and aircraft noise based on FAA regulations. The sketch in Figure 3 illustrates this notion for a simple example with two design variables and five constraints. The whole square denotes the design space, i.e. all possible design variable setting combinations. The dark lines mark the constraints as functions of the design variables for a particular constraint value that needs to be satisfied. The white area in the middle denotes the feasible space.

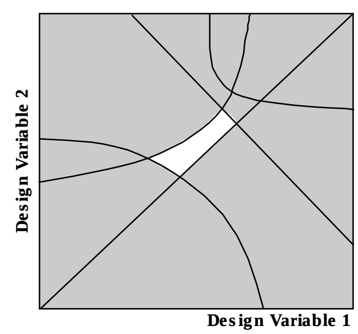

Figure 3: Feasible Space in a Design Space

The computation of feasibility as formulated deterministically requires an exorbitant amount of work, which can be reduced through the use of probability theory. Such a formulation assumes uniform distributions for the design variables, i.e. assigning each possible variable value the same

[***] The main difference between a design solution and design concept lies in the level of determination of the design variables. It's a solution, if all variable settings are known. It's a concept, if only few settings are determined, but the products major functionalities and components are known.[18]

probability of occurrence. The problem reduces to determining the probability of satisfying all constraints concurrently. This probability is equivalent to the volume spanned over the feasible area, since the uniform distributions introduce a constant height over the area, and is therefore a measure for feasibility. This process is also often referred to as the *system reliability problem*. With the probabilistic process at hand, and recognizing that the constraints are simply random variables, a joint probabilistic approach can be used to determine system feasibility.

This technique is very useful in conceptual and preliminary design, where concept feasibility needs to be evaluated quickly in order to determine whether a particular concept should enter the next design phase. For example, if little feasibility is found, new technologies may be introduced to the design concept in hope of increasing the feasible space. Through repeated execution of this method, different technologies can be applied to the system, while a growth or shrinkage of feasible space manifests their benefit. For further discussion of feasibility studies, please refer to References 12 and 13.

The proposed High Speed Civil Transport aircraft is a particularly good example for this method, since it is a next generation aircraft that has very little chance of satisfying all constraints with today's technology. In other words, the HSCT concept at today's technology levels has a very small feasible space, if any at all. Since the feasibility study is just an application for the joint probability formulation introduced in this paper, only a simple example for two constraints, approach speed and take-off field length, is executed here. Specifically, noise constraints are not considered here, since they cannot be satisfied with the simulated current technology. The design variables used are listed in Table II. They represent some of the key drivers in airplane design.[11] Uniform distributions over their indicated ranges are assigned to all variables. To illustrate the kink location, a notional HSCT planform is depicted in Figure 4.

Table II : Design Variable Description and Range

Variable	Name	Range
Thrust to Weight Ratio	TWR	0.28 - 0.32
Wing Area	WingArea	$8.5 - 9.5 * 10^3\ ft^2$
Longitudinal Kink Location	x1	1.54 - 1.62
Spanwise Kink Location	y1	0.5 - 0.58
Turbine Inlet Temperature	TIT	$3 - 3.25 * 10^3$ degF
Fan Pressure Ratio	FPR	3.5 - 4.5

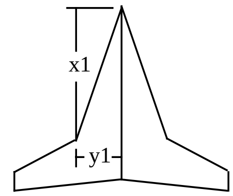

Figure 4: Illustration of the Kink Location

A Monte-Carlo simulation, employing the uniform distributions for the variables in Table II, is used to generate 10,000 outputs for take-off field length and approach speed from the aircraft synthesis/sizing code FLOPS.[19] The outputs are collected in pairs and are thus jointly distributed. The empirical distribution function (Equation (15)), derived from the data set, is illustrated in Figure 5. It yields for the constraint values of 10,500 ft for take-off field length and 154 kts for approach speed:

$$P(TOFL \le 10{,}500\,ft, VAPP \le 154kts) = F(10{,}500\,ft, 154kts)$$
$$= \tfrac{1}{10{,}000}\sum_{i=1}^{10{,}000} I(a_{iTOFL} \le 10{,}500\,ft, a_{iVAPP} \le 154kts) = 0.0107$$

In other words, the chance of finding a feasible HSCT design within the specified design space is 1.07%, based on the EDF method.

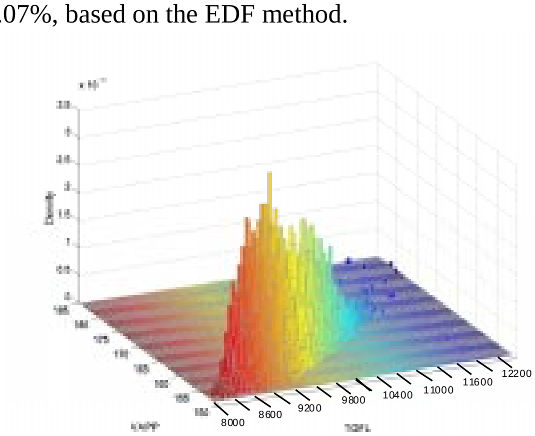

Figure 5: Joint Probability Distribution for Take-Off Field Length and Approach Speed Based on the EDF Method

To demonstrate the Joint Probability Model, the sample mean and standard deviation as well as the correlation coefficient for take-off field length and approach speed are taken from the sample data. This yields a mean of 9,692.2ft for take-off field length, with a standard deviation of 510.5ft, and a mean of 160.3kts for approach speed, with a standard deviation of 2.9997kts. The correlation coefficient is estimated to be 0.7311. The joint distribution with these statistics is depicted in Figure 6. Using these parameters, the numerically integrated joint probability model (Equation (11)) estimates the feasibility to be:

$$P(TOFL \le 10{,}500\,ft, VAPP \le 154kts) = F(10{,}500\,ft, 154kts) =$$

$$\int_0^{154}\int_0^{10{,}500} \frac{1}{2\pi 510.5 \cdot 3\sqrt{1-0.7311^2}} \exp\{\frac{1}{2\cdot 0.7311^2 - 2}[(\frac{tofl-9692.2}{510.5})^2$$

$$-2\cdot 0.7311(\frac{tofl-9692.2}{510.5})(\frac{vapp-160.3}{2.9997}) + (\frac{vapp-160.3}{2.9997})^2]\} dtofl \cdot dvapp$$

$$= 0.0153$$

Hence, the feasibility of this HSCT concept with today's technology is estimated as 1.53%, based on the JPM method.

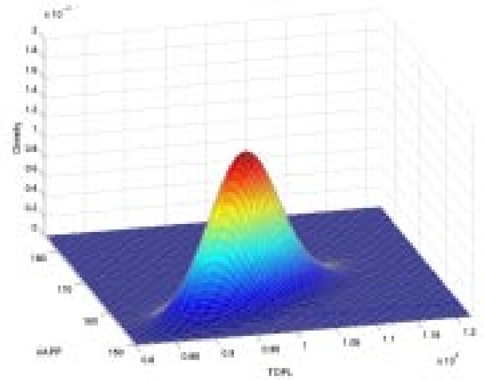

Figure 6: Joint Probability Distribution for Take-Off Field Length and Approach Speed Based on the Joint Probability Model

To illustrate the difference in the joint probability distribution generated by the EDF method and the Joint Probability Model, levels of constant probability as a function of take-off field length and approach speed are plotted for both methods in Figure 7. The ellipses are rings of constant probability density based on the JPM method. The scattered lines are lines of constant probability density based on the EDF method. The area of interest has also been marked in the plot to indicate how well the HSCT concept studied here satisfies the constraints. The volume underneath the distribution function within the area of interest is equal to the feasibility value calculated by the two methods.

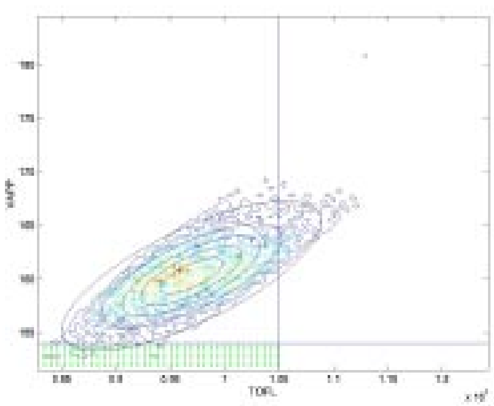

Figure 7: Comparison of Probability Ellipses from the EDF and JPM Methods

As the similarities of the ellipses in Figure 7 indicate, both methods predict the joint probability distribution equally well. Therefore, it appears both methods are well suited for solving the feasibility problem, and choosing one over the other depends only on the type of criterion information available, i.e. data or marginal distributions and correlation coefficient.

The Viability Problem

The viability problem is a typical example of a multi-criteria decision making problem. The example employed here involves two conflicting criteria: the aircraft price the manufacturer needs to charge to make a profit and the airline return on investment (ROIA). Common sense suggests that if the price increases the ROIA should decrease, given constant markets and average yield. However, the aircraft will have to satisfy base levels for the price and the airline ROI. A joint probabilistic method can not only estimate the probability of satisfying the base levels, but also allow one to visualize the trades made in a decision making process. The probability of satisfying those criterion base levels is called *viability*.

The HSCT concept is employed once again for the viability study. In this instance, the configuration is fixed, i.e. the aircraft does not change. The changing parameters here are economic variables of interest to the manufacturer as well as the airline. They have been summarized in Table III.

Table III: Econimic Variable Descriptions and Values

Variable	Name	Range	Distribution Type	Mode
Learning Curve	LC	0.8 – 0.9	Triangular	0.85
Production Quantity	ProdQ	300 – 800	Triangular	650
RDT&E Complexity	RDTE$	-15% - +15%	Triangular	-.05
Load Factor	LF	0.65 – 0.85	Triangular	0.8
Economic Range	ECR	3000 – 5000	Triangular	3200
Fuel Cost	Fuel$	0.60 – 1.20	Triangular	0.7

All variables are inputs to the cost estimation program used, called ALCCA.[20] Learning Curve changes all manufacturing learning curves. RDTE$ is a complexity factor for the RDT&E cost, which simulates improvements made to the design cycle time. Load Factor is the ratio of equivalent full fare booked seats to the number of available seats, and Economic Range is the distance between city pairs the HSCT is scheduled to connect. For simplicity, all variables have triangular distributions with range and mode as indicated in Table III.

Similarly to the feasibility problem, 10,000 paired sample points for price and ROIA are generated with a Monte-Carlo simulation, yielding a joint probability distribution through the EDF method, depicted in

Figure 8. Input distributions for the Monte Carlo simulation are listed in Table III. Viability for the concurrent achievement of \$275M and 12% for ROIA can then be calculated as:

$$P(\text{Price} \geq \$275M, \text{ROIA} \geq 12\%) = F(\$275M, 12\%) =$$
$$\tfrac{1}{10{,}000} \sum_{i=1}^{10{,}000} I(a_{i\text{Price}} \geq \$275M, a_{i\text{ROIA}} \geq 12\%) = 0.0204$$

Thus, the viability of this HSCT concept is 2%, based on the EDF method.

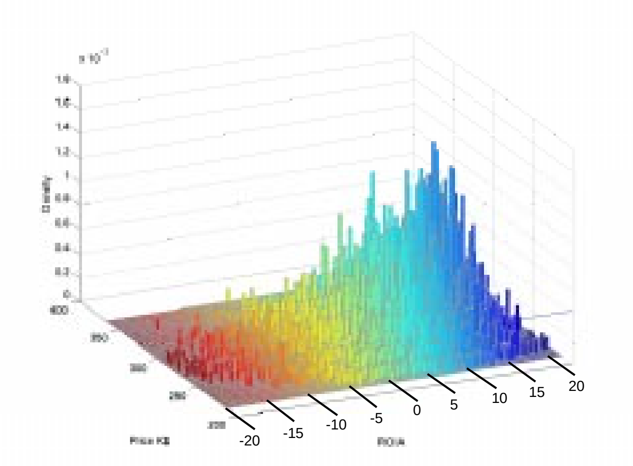

Figure 8: Joint Probability Distribution for Price and ROIA Based on the EDF Method

Again, the samples are used to estimate the mean, standard deviation, and correlation coefficient to verify the applicability of both joint probabilistic methods. The samples yield a mean of \$270,105M for the manufacturer's price, with a standard deviation of \$24.544M, and a mean of 5.745% for the airline's ROI, with a standard deviation of 6.134%. The correlation coefficient is estimated to be –0.208. The correlation coefficient is as expected negative, indicating a conflict in maximizing both criteria. Using the statistics from above, a joint probability distribution can be created, depicted in Figure 9, and the viability for base levels of \$275M for price and 12% for ROIA can be calculated as:

$$P(\text{Price} \geq \$275M, \text{ROIA} \geq 12\%) = F(\$275M, 12\%) =$$
$$\int_{275}^{\infty}\int_{12\%}^{\infty} \frac{1}{2\pi \cdot 24.5 \cdot 6.1\%\sqrt{1-0.208^2}} \exp\{\frac{1}{2 \cdot 0.208^2 - 2}[(\frac{price-270.1}{24.5})$$
$$-2 \cdot 0.208(\frac{price-270.1}{24.5})(\frac{roia-5.7\%}{6.1\%})$$
$$+(\frac{roia-5.7\%}{6.1\%})^2]\} droia \cdot dprice = 0.0473$$

Hence, the viability of this HSCT concept is estimated with 4.7%, based on the joint probability model.

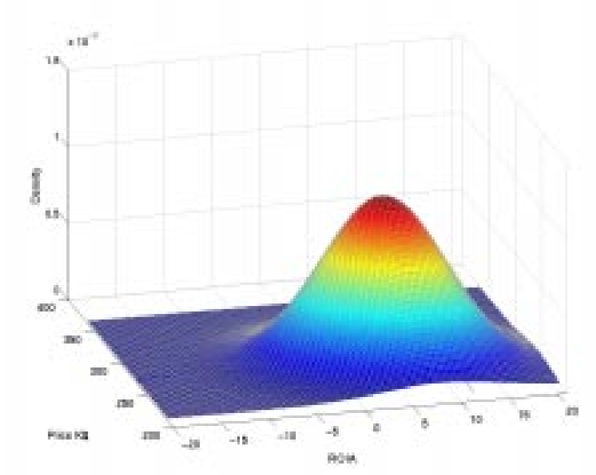

Figure 9: Joint Probability Distribution for Price and ROIA on the Joint Probability Model

A comparison of the probability contours from the JPM with the contours from the EDF in Figure 10 indicates, both methods are appropriate for the viability problem. The area of interest, displayed in Figure 10, marks the area of acceptable values for the price and the airline's ROI. The ellipses are rings of constant probability density based on the JPM method. The scattered lines are lines of constant probability density based on the EDF method, which suggest together with the histogram in Figure 8 that the joint distribution is slightly skewed towards higher ROIA values. Consequently, the normality assumption made here for the JPM may not be satisfied, thus yielding a higher viability than the EDF method. It is thus highly recommended to extend the JPM to include more distributions than the normal.

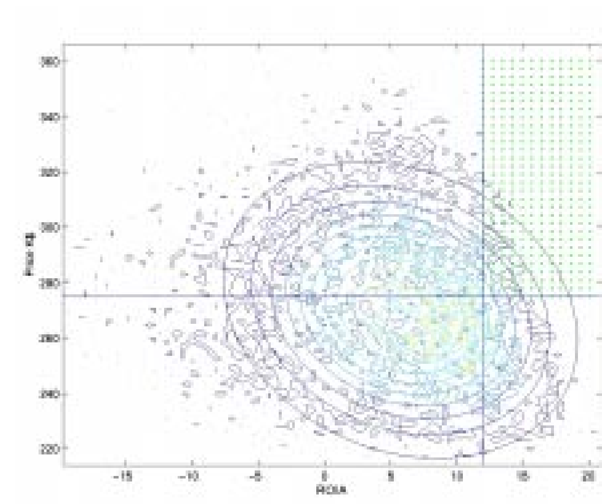

Figure 10: Comparison of Probability Ellipses from the EDF and JPM Methods

A more valuable plot for the decision making trade-offs is the joint cumulative probability plot, depicted in Figure 11 and generated with the EDF method. The lines of constant cumulative probability indicate what levels of price and ROIA can be satisfied for a given probability. Cumulative probability increases for smaller values of price and ROIA. Since both criteria have conflicting goals, trade-offs can be made as to: “what price can I charge as a manufacturer, with what certainty, and how much ROI does the airline have to sacrifice?” These questions can best be answered in the ‘knee’ area of the probability lines. The certainty is constant on the line and by following it, one can easily observe how much one criterion has to give to allow for larger values of the other. Note also that there are limits as to what price the manufacturer can charge or the airline ROI can achieve for a given probability, regardless of the value for the other criterion. The straight lines of constant probability indicate these limits. They simply represent the univariate cumulative distribution functions that indicate the chance of satisfying particular criterion levels, regardless of any other criteria. The cumulative probability plot displays very clearly how misleading this assumption can be. Satisfactory levels for one criterion may have a large chance of yielding highly undesirable levels for the others. Furthermore, the cumulative probability plot displays the current viability estimate, indicated by the crossbars, and how viability changes, if the base levels were to change. In traditional viability analyses new base levels always meant a new execution of the viability estimation. The multivariate joint probabilistic method yields this information the first time through, thus helping to reduce design cycle time.

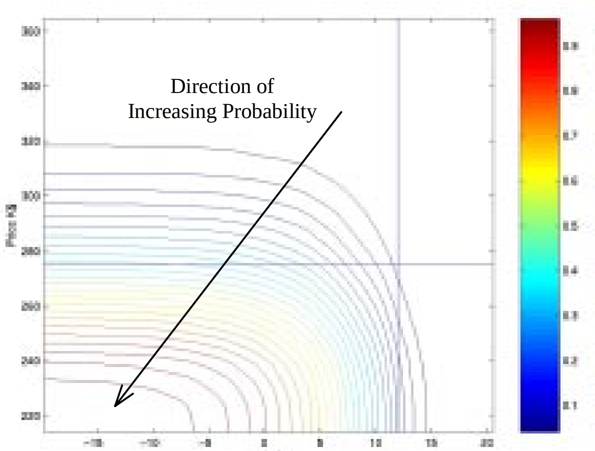

Figure 11: Cumulative Probability Plot for Price and ROIA

Conclusions

As demonstrated in this paper, both methods, the Joint Probability Model and the Empirical Distribution Function, are appropriate for calculating the joint probability of multiple criteria in conceptual and preliminary design. Their selection depends mainly on the availability of data. If large amounts of data can be produced with reasonable effort, the empirical distribution function is a more accurate approach to calculating the joint probability. If data on the criteria is not available or too expensive to obtain, the joint probabilistic model is an appropriate way of generating information about their joint probabilistic behavior. An extension of the model, currently pursued by the authors, including distributions other than the normal would further enhance its prediction accuracy. This broad usability of the joint probabilistic formulation of a multi-criteria decision making makes this method applicable to several phases in design, ranging from early conceptual (no data) to hardware testing (exact data). It can thus answer questions about feasibility and viability of design concepts and solutions. The paper also suggests the joint probability of success as an objective function for the field of Multi-Disciplinary Optimization.

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